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Applicability of an analytical solution for ground settlement induced by circular tunnel

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ABSTRACT

The influences of the free surface of a shallow tunnel should be considered to estimate the ground settlement accurately. An analytical solution is presented to describe the ground deformation induced by circular tunnel in an elastic half-plane. The solution uses the complex variable method with a conformal mapping onto a circular ring. The coefficients in the Laurent series expansion of the stress functions can be obtained from the boundary conditions. The applicability of the analytical solution is checked with five case studies. Generally good agreement of the predicted ground deformations can be seen with field observations for tunnels in stiff clay. Also, it is assumed to have a good estimation for tunnels in rock mass. The analytical solution is meaningful on understanding the ground behavior with some level of accuracy.

1. INTRODUCTION

Ground deformation induced by tunnelling may result in surface settlements. In engineering practice the surface settlements are described by empirical formulas based upon field observations such as the Gaussian probability curve proposed by Peck (1969). However, they cannot be applicable to different ground conditions or construction methods, and they cannot give an information about horizontal movements or subsurface settlements. In this manner, a variety of analytical solutions have been proposed for estimating the two-dimensional distribution of ground movements. Although they are somewhat poor schematization of the ground behaviour, an elastic solution may well serve to investigate the characteristics of the stress and displacement field.

2. COMPLEX VARIABLE SOLUTION FOR GROUND DEFORMATION

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Complex variable method presented by Muskhelishvili (1953) and Sokolnikoff (1956) is particularly suitable for a shallow tunnel as it allows mapping the domain to describe both boundaries, excavated circular tunnel and ground surface, by a single coordinate. Verruijt (1997, 1998) successfully extended the method to solve the shallow tunnel problem in an elastic half-plane by using a conformal mapping on a circular ring and a representation of the complex potentials as Laurent series. In conclusion, the displacements and stresses are expressed as below,

$$\frac{\partial \psi}{\partial x} + i\frac{\partial \psi}{\partial y} = \phi(z) + z\overline{\phi}'(z) + \overline{\chi}'(z)$$

$$2\mu\left(u_x + iu_y\right) = \kappa\phi(z) - z\overline{\phi}'(z) - \overline{\chi}'(z)$$

$$\kappa = \frac{\lambda + 3\mu}{\lambda + \mu} = 3 - 4\upsilon$$

In this study, elastic half plane in the complex z-plane is mapped onto a ring in the ζ-plane, bounded by the circles |ζ|=1 and |ζ|=α, where α<1. The transformation function ω is expressed as

$$z = \omega(\zeta) = -ih\frac{1-\alpha^2}{1+\alpha^2}\frac{1+\zeta}{1-\zeta}, \left(\frac{r}{h} = \frac{2\alpha}{1+\alpha^2}\right)$$

where r is the excavation radius, h is the depth of the cavity center, and α is a parameter defined by the ratio of the radius and the depth.

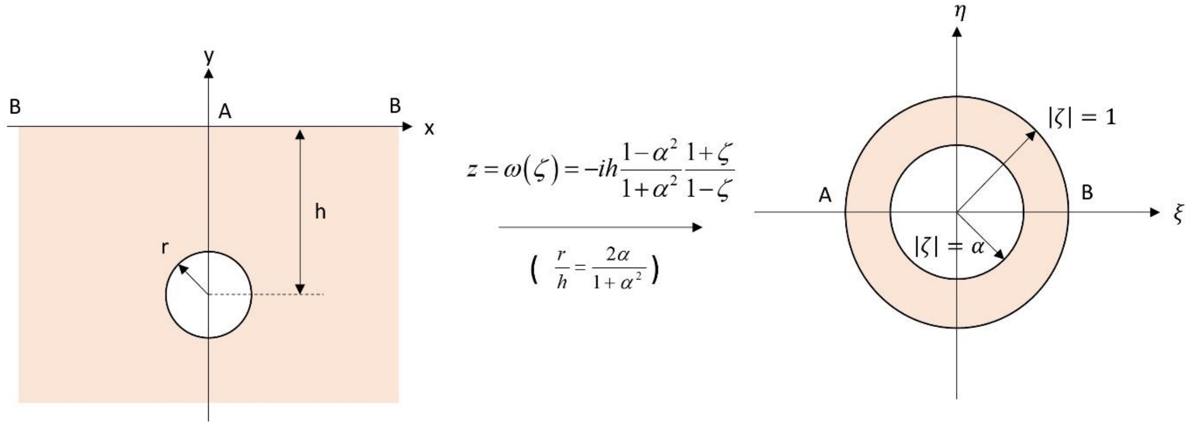

Figure 1. Conformal mapping from half-plane to annulus.

Because the conformal transformation function is analytic in the ring bounded by the inner and outer circles, the analytic functions from complex variable method ϕ(z) and χ'(z) can be considered as functions of ζ, and they are both analytic in the region in the ζ-plane. Those complex analytic functions can be represented by Laurent series expansions,

$$\phi(\zeta) = a_0 + \sum_{k=1}^{\infty} a_k \zeta^k + \sum_{k=1}^{\infty} b_k \zeta^{-k}$$

$$\chi'(\zeta) = c_0 + \sum_{k=1}^{\infty} c_k \zeta^k + \sum_{k=1}^{\infty} d_k \zeta^{-k}$$

From the boundary condition of circular excavated surface, all of the coefficients of Laurent series can be determined and result in the stress and displacement field of elastic half-plane. The boundary condition should be specified before calculation as a form of deformation of traction force.

3. RESULTS AND DISCUSSIONS

Five case histories, previously investigated by various authors, have been selected in this study to evaluate the analytical prediction and the ground deformation behaviours. Table 1 summarizes the typical values of the tunnel geometry, ground properties, and gap parameters estimated using the theoretical methods proposed by Lee et al. (1992) for all cases, used in former study. Generally, former studies discussed about analytical solutions adopting the Poisson's ratio of 0.495 considered the ground as the short-term, undrained. However, the field observations used for comparison are gathered at some time after the tunnel was excavated. The Poisson's ratios obtained by laboratory testing are used; 0.2 for Heathrow Express Trail Tunnel and Green Park Tunnel, 0.3 for Barcelona Subway Network Extension, 0.33 for Thunder Bay Tunnel, and 0.2 for Bangkok Sewer Tunnel.

Table 1. Geometry and ground elastic properties for case studies

Tunnel	h/r [m]	E [MPa]	Poisson's ratio [-]	Unit weight [kN/m^3]	Gap [mm]	Soil type
Heathrow Express Trial	19/4.25	33	0.2	19	58	Stiff London clay
Green Park	29.4/2.07	40	0.2	19	34	London clay
Barcelona Subway Network Extension	10/4	25	0.3	18	31	Stiff clay with gravel
Thunder Bay	10.7/1.235	10	0.33	18	164	Soft silty clay
Bangkok Sewer	18.5/1.33	20	0.2	17	81	Soft to stiff clay

Obtained and predicted surface settlements for Heathrow Express Trial tunnel and the Bangkok Sewer tunnel are presented below.

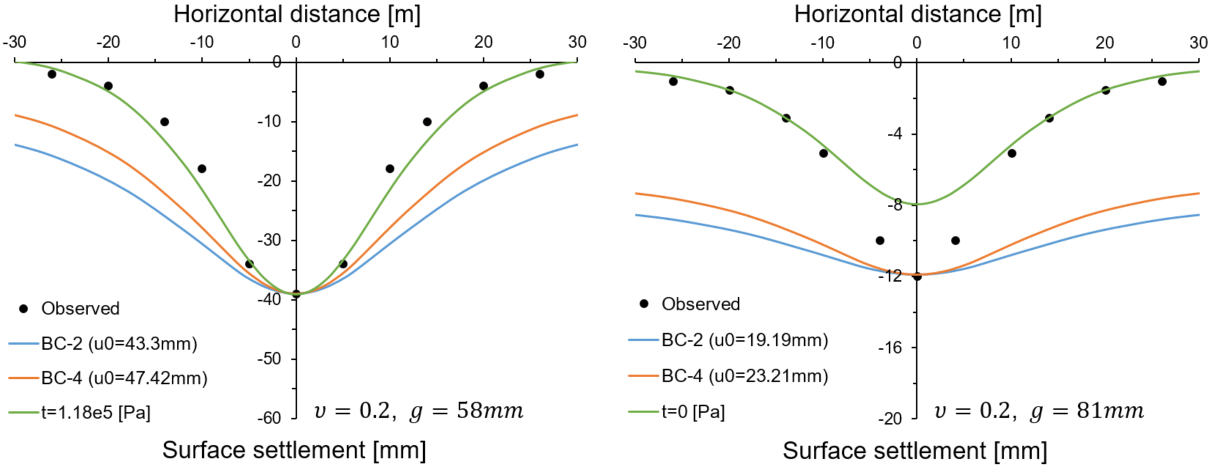

Figure 2. Observed and predicted surface settlement. (Left) Heathrow Express Trial Tunnel; (Right) Bangkok Sewer Tunnel.

4. CONCLUSIONS

The stress-based solution is suitable for describing the ground deformation behavior and is good at prediction for proper construction through the rock mass or stiff clay which show less plastic region and deformed as homogenous material. The displacement-based solution is relatively not proper for simulating the ground deformation behavior due to its controlled deformation pattern, but if the ground loss can be clearly assessed, it can help to derive the maximum settlement considering the construction factors other than geotechnical factors.

ACKNOWLEDGEMENTS

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